# Design of PEXIT Algorithm for SIMO Rayleigh Fading Channel 

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#### Abstract

In this paper we propose a modified PEXIT algorithm for SIMO Rayleigh fading channel. In this paper we first explain PEXIT algorithm which shows outstanding performance over additive white Gaussian noise channels but they are not giving accurate results for SIMO Rayleigh fading channel. Then we modified PEXIT algorithm so that it will give accurate results for both the channels.

Index Terms- Extrinsic information transfer, Protograph Extrinsic information transfer, Log Likelihood ratio, Maximum ratio combining, Equal gain combining.


## 1. INTRODUCTION

Many LDPC codes[1] are irregular and suffer from high error floor and nonlinear encoding. So another novel class of LDPC code, namely multiedge type (MET) LDPC code, has been introduced [2] whose subclass the protograph- based LDPC code has emerged as a promising FEC scheme due to its excellent error performance and low complexity [3]. Extrinsic information transfer (EXIT) [4] charts are used as a density evolution technique for the decoding convergence analysis of iterative decoders. EXIT charts have been used in [5] to design low-density parity-check (LDPC)codes due to their simplicity and accuracy in performance prediction. But EXIT charts are not applicable for the protograph based [6] LDPC codes.

In this paper, we aim to investigate the performance of the protograph codes over a SIMO Rayleigh fading channel. To do so, first we study the assumption of PEXIT algorithm and then we propose a modified PEXIT [7] algorithm for analyzing the protograph LDPC code over a fading environment. In this paper we consider sixteen receiver antennas. We have also studied probability density factor(PDF). At the receiver, we assume that received signals are combined using the maximum-ratiocombining (MRC) method [8] or the equal-gaincombining (EGC) method. [9]

## 2. ASSUMPTION OF THE PEXIT ALGORITHM

A protograph EXIT (PEXIT) algorithm has been used for the analysis and design of protograph codes over the AWGN channel. In this paper first we illustrate the PEXIT algorithm, which works well on the AWGN channel. This algorithm is not useful for the SIMO Rayleigh fading channel. Then we modify the PEXIT algorithm for such a channel and use it for analyzing the protograph codes in our system.

An important assumption of the proposed PEXIT algorithm in [10] is that the channel log-likelihoodratio (LLR) messages should follow a symmetric Gaussian distribution. In this paper, we briefly illustrate that this assumption cannot be maintained in the case of a SIMO Rayleigh fading channel and then we elaborate how to apply the PEXIT algorithm in such an environment. To simplify the analysis, we assume that the all-zero codeword is transmitted.

### 2.1. System Model

In this paper we consider the system where firstly the information bits are punctured by the protograph LDPC code. Then the binary coded bits $v \in\{0,1\}$ are passed to a binary-phase-shiftkeying (BPSK) modulator, the output of which is given by $x=(-1) v \in\{+1,-1\}$. The modulated signal $x$ is further sent through a SIMO fading channel with one transmit antenna and $N_{R}$ receive antennas.
We denote $h$ as a channel realization vector of size $N_{R} \times 1$, the entries of which are complex independent Gaussian random variables with zeromean and variance $1 / 2$, i.e., $N(0,1 / 2)$, per dimension. Then, the $N_{R} \times 1$ receive signal vector, denoted by r , is given by

$$
\begin{equation*}
r=h x+n \tag{1}
\end{equation*}
$$

Now by using $j(j=1,2, \ldots)$ to indicate the coded bit number and $k\left(k=1,2, \ldots, N_{R}\right)$ to indicate the receive antenna number, the signal of the $j^{T H}$ coded bit at the $k^{T H}$ receive antenna can be written as,

$$
\begin{equation*}
r_{j}[k]=h_{j}[k] x_{j}+n_{j}[k] \tag{2}
\end{equation*}
$$

In this paper, we assumed interference does not exist in the channel. So, we apply the simpler MRC and EGC [8],[9], which have also been used to process the received signals over interference-free Rayleigh
fading channels incorporated with multiple antennas and LDPC codes [11],[12].
The combiner output corresponding to the $j^{T H}$ coded bit, denoted by $y_{j}$, is then given by [8], [9].

$$
\begin{align*}
& y_{j}=\sum_{k=1}^{N_{n} h_{j}^{*}[k] \eta_{j}[k] \quad \text { for MRC }}  \tag{3}\\
& y_{j}=\sum_{k=1}^{N_{n}} \frac{\hat{h}_{j}^{\prime}[k]}{\left.h_{j}[k]\right]} \eta_{j}[k] \text { for EGC } \tag{4}
\end{align*}
$$

where * denotes the complex conjugate, $|$. represents the modulus operator and $h_{j}^{*}[k] \backslash h_{j}[k] \mid$ is used to remove the phase ambiguity for coherent reception in EGC. Here $y_{j}$ is calculated by combining MRC and EGC values.

### 2.2. Channel LLR values

Now we calculate initial channel LLR value which we will denote by $L_{c h, j}$ corresponding to the $j^{T H}$ coded using [4].

$$
\begin{align*}
L_{c h, j} & =\operatorname{In}\left(\frac{\operatorname{rr}\left(y_{j}=0 \mid y_{j} h_{j}\right]}{\operatorname{Fr}\left[v_{j}=0 \mid y_{j} \hat{h}_{j}\right]}\right)  \tag{5}\\
& =\operatorname{In}\left(\frac{\operatorname{Pr}\left[x_{j}=+1 \mid y_{j} h_{j} j\right.}{\operatorname{Pr}\left[x_{j}=-1 \mid y_{j} h_{j} j\right.}\right) \tag{6}
\end{align*}
$$

$$
L_{c h, j}= \begin{cases}\frac{2 y_{j}}{\sigma_{n}^{2}} & \text { for MRC }  \tag{7}\\ \frac{y_{j}}{N_{n} \sigma_{1}^{2}}\left(\sum_{k=1}^{N_{N}} \| h_{j}[\kappa] \mid\right) \quad \text { for EGC }\end{cases}
$$

where $\operatorname{Pr}(\cdot)$ is the probability function and $h_{j}=\left[h_{j}\right.$ $\left.[1], h_{j}[2], \cdots, h_{j}\left[N_{R}\right]\right]^{T}$ (here superscript " T " represents the transpose operator). In next stage will put the values from "Eq.(3)"and "Eq.(4)" into the "Eq.(7)" so that will get the LLR value of both MRC and EGC.
"Eq.(8)"gives us LLR value for the two combiners. Further the performance of the two combiners is calculated by exploiting Monte Carlo simulations i.e., by generating a large number of independent channel realizations and computing their average capacity value. For that, we consider a SIMO Rayleigh fading channel with $N_{R}=16$ and $E_{b} / N_{0}=6.0209 d B$. By sending $x_{j}=+1$ repeatedly while varying the channel fading vector $h_{j}$ from bit to bit, we evaluate the mean of the absolute value of $L_{\text {ch. }}$. We observe that the MRC produces an average value of 18.2013 , i.e., $\mathrm{E}_{\mathrm{MRC}}\left(\left|L_{\text {ch } \mathrm{H}_{j}}\right|\right)=18.2013$ whereas the EGC gives $\mathrm{E}_{\mathrm{EGC}}\left(\left|L_{\text {chyj }}\right|\right)=4.3057$. As MRC provides a higher
variance than EGC will focus on MRC for successful decoding.
We denote the real part of $L_{c h, j}$ by "L(real)". In Fig. 1, we further plot the probability density function (PDF) of the $L$ (real) values denoted by" frequency of $L$ (real) " when MRC is used. The curves in the figure indicate that the PDF of the Lre, j values does not follow a symmetric complex Gaussian distribution. This clearly shows that the PEXIT algorithm in [13] is not applicable to this type of channel. By observing Fig. 1.we conclude that the channel LLR values for a SIMO fading channel do not follow a symmetric Gaussian distribution and hence the PEXIT algorithm cannot be applied directly.
In the following, we analyze the distribution of the $L_{c h, j}$ values when the channel realization is fixed. We consider a fixed channel realization, i.e., a fixed channel fading vector $h_{j}$. We assume using the allzero codeword (i.e., $x_{j}=+1$ rep) and we substitute (2) into (8). Then, we can rewrite the expression for $L_{c b_{j} j}$ as

$$
\begin{align*}
L_{c h, j} & =\frac{2}{a_{n}^{2}} \sum_{\mathbb{k}=1}^{N_{n}} h_{j}^{*}[k]\left(h_{j}[k] x_{j}+n_{j}[k]\right) \\
& =\frac{2}{a_{n}^{2}} \sum_{k=1}^{N_{k}}\left(\mid h_{j}\left[\left.k\right|^{2}+h_{j}^{*}[k] n_{j}[k]\right)\right. \tag{9}
\end{align*}
$$

From "Eq.(9)" we can write channel factor $\alpha_{j}$ as

$$
\begin{equation*}
\alpha_{j}=\sum_{\tilde{\pi}=1}^{N N_{1}}\left\|h_{j}[k]\right\|^{2} \tag{10}
\end{equation*}
$$

and $\sigma_{12}^{2}$ is calculated by using equation written below

$$
\begin{equation*}
\sigma_{\mathrm{n}}^{2}=\frac{N_{B}}{2 \pi\left(C_{\mathrm{E}} / N_{0}\right)} \tag{11}
\end{equation*}
$$

where, $R$ is nothing but code rate and $E_{b} / N_{0}$ is SNR of the system. So by using these expressions we calculate LLR values which follow symmetric complex Gaussian distribution for a fixed fading vector. Using this property, we propose a modified PEXIT algorithm that can be adopted to analyze the protograph codes.

## 3. MODIFIED PEXIT ALGORITM FOR SIMO RAYLEIGH FADING CHANNEL

Here first we define some symbols and terms which we are used in our algorithm. A protograph $G$ $=(V, C, E)$ consists of three sets $V, C$ and $E$ corresponding to the variable nodes, check nodes and edges, respectively [6]. In a protograph, each edge $\theta_{i j j} \in \mathrm{E}$ connects a variable node $v_{j} \in \mathrm{~V}$ to a check node $\sigma_{i} \in \mathrm{C}$. Moreover, parallel edges are allowed.
A large protograph (namely a derived graph) corresponding to the protograph code can be obtained by a "copy-and-permute" operation. Hence, codes with different block lengths can be generated by performing the "copy-and-permute" operations different number of times.

A protograph with $N$ variable nodes and $M$ check nodes can be represented by a base matrix $B$ of dimension $M \times N$. The $(i, j)^{t h}$ element of B , denoted by $b_{i d}$, represents the number of edges connecting the

Now, for a rate- $R$ protograph with $N$ variable nodes and $M$ check nodes, the proposed modified PEXIT algorithm over a SIMO Rayleigh fading channel can be described as follows.


Fig. 1. Probability density functions of the L(real) values over the SIMO Rayleigh fading channel.
variable node $v_{j}$ to the check node $c_{\mathrm{i}}$. Here we define five types of mutual information (MI) as follows.

- $\quad_{A v}(i, j)$ denotes the a priori MI between the input LLR value of $v_{i}$ on each of the $b_{i, i}$ edges and the corresponding coded bit $v_{i}$.
- $I_{A c}(i, j)$ denotes the a priori MI between the input LLR value of $c_{1}$ on each of the $b_{i j}$ edges and the corresponding coded bit $v_{j}$.
- $I_{E v}(i, j)$ denotes the extrinsic MI between the LLR value sent by $\vartheta_{j}$ to $c_{i}$ and the corresponding coded bit $v_{j}$.
- $I_{E c}(i, j)$ denotes the extrinsic MI between the LLR value sent by $c_{i}$ to $v_{j}$ and the corresponding coded bit $v_{j}$.
- $I_{a n a}(j)$ denotes the a posteriori MI between the a posteriori LLR value of $v_{i}$ and the corresponding coded bit $v_{i}$.
In addition, during each iteration in the PEXIT algorithm, we have $I_{A v}(i, j)=I_{E v}(i, j)$ and $I_{A v}(i, j)=$ $I_{E}(i, j)$. We also denote the maximum number of iterations in the algorithm by $T_{\max }$. Besides, we define two new terms called indicator function and punctured label.
Definition 1. We define the indicator function $\varphi(\cdot)$ of an element $b_{i j}$ in the base matrix B as

$$
\varphi\left(b_{i j}\right)=\left\{\begin{array}{cc}
1 & \text { if } b_{i j} \neq 0  \tag{12}\\
0 & \text { otherwise }
\end{array}\right.
$$

Hence, $\varphi\left(b_{i, j}\right)$ indicates whether $v_{j}$ is connected to $c_{i}$ or not.
Definition 2. We define the punctured label $P_{j}$ of a variable node $v_{j}$ as 0 if $v_{j}$ is punctured, and 1 otherwise.
(1) For a given SIMO channel realization $\mathrm{h}=\left[h_{j}\right.$ [1], $\left.h_{j}[2], \cdots, h_{j}\left[N_{R}\right]\right]^{T}$, we can calculate the corresponding channel factor $\alpha$ using (10), i.e., $\alpha=\sum_{k=1}^{N_{R}} \|\left. h_{j}[k]\right|^{2}$. Suppose we are given the number of blocks of channel factors (denoted by $Q$ ) and the maximum number of iterations $\left(\mathrm{T}_{\max }\right)$. We generate a matrix $\alpha=\alpha_{q_{j}}=$ $\sum_{k=1}^{N_{n}}\left\|h_{q, j}[k]\right\|^{2}$ of dimension $Q \times N$ to represent the $Q$ blocks of channel factors, i.e., each row in $a$ represents a group of channel factors for the $N$ variable node in the protograph. We also select an initial $E_{b} / N_{0}$ (in dB ) which should be sufficiently small.
(2) For $i=1,2, \ldots, M$ and $j=1,2, \ldots, N$, we set the initial $I_{A v}(i, j)$ to 0 . We also reset the iteration number $t$ to 0 . Considering the punctured label and substituting "Eq.(11)"into "Eq.(9)", for the channel factor ${\widetilde{u_{q, j}}}(j=1,2, \ldots$ $, N$ and $q=1,2, \ldots, Q)$, the corresponding variance of the initial LLR value(denoted by $\left.\sigma_{c b_{i} q_{j}}^{2}\right)$ is given by

$$
\begin{align*}
\sigma_{W_{q j}}^{2} & =\frac{4 P_{j} \alpha_{q_{d j}}}{\sigma_{12}^{2}} \\
& =\frac{\operatorname{snf}_{j} \alpha_{4 j}}{N_{R}} 10^{\frac{\left(E_{2} / N_{a}\right)}{10}} \tag{13}
\end{align*}
$$

(3) If $t=T_{\max }$, set $E_{b} / N_{0}=E_{b} / N_{0}+0.001 \mathrm{~dB}$ and go to Step 2; otherwise, for $i=1,2, \ldots, M ; j=1,2$, $\ldots, N$ and $q=1,2, \ldots, Q$, we calculate output extrinsic MI sent by $v_{j}$ to $c_{i}$ for the $q^{\text {th }}$ fading block.
(4) For $i=1,2, \ldots, M$ and $j=1,2, \ldots, N$, we obtain the expected value of $I_{E v, q}(i, j)$ using

$$
\begin{equation*}
I_{A c}(i, j)=E\left[I_{E v, q}(i, j)\right] \tag{14}
\end{equation*}
$$

(5) For $i=1,2, \ldots, M$ and $j=1,2, \ldots, N$, we compute the output extrinsic MI sent by $c_{i}$ to $v_{j}$. Then, we get the a priori MI between the input LLR of $v_{j}$ on each of the $b_{i, j}$ edges and the corresponding coded bit using

PEXIT algorithm for SIMO Rayleigh fading channel. Furthermore, we can explore extending the modified PEXIT algorithm to other systems such as the MIMO fading systems.


Fig. 2 Signal to noise ratio $\left(E_{B} / N_{O}\right)$ Vs. Bit error rate(BER) using modified PEXIT algorithm for SIMO fading channel

$$
\begin{equation*}
I_{A v}(i, j)=I_{E c}(i, j) \tag{15}
\end{equation*}
$$

(6) For $j=1,2, \ldots, N$ and $q=1,2, \ldots, Q$, we compute the a posteriori MI of $v_{j}$. Then, for every $j=1,2, \ldots, N$, we can evaluate the expected value of $I_{a p p, q}(j)$ using

$$
\begin{equation*}
E\left[I_{a p p, q}(j)\right]=\frac{1}{q} \sum_{q=1}^{Q} I_{a p p, q}(\hat{}) \tag{16}
\end{equation*}
$$

(7) If the expected MI values $E\left[I_{a p p, q}(j)\right]=1$ for all $j$ $=1,2, \ldots, N$, the $E_{b} / N_{0}$ value will be the EXIT threshold that allows all variable nodes to be decoded correctly and the iterative process is stopped; otherwise, we increase $t$ by 1 and go to Step 3 to continue the iterative process.
To maintain the accuracy of the modified PEXIT algorithm we have to follow all these steps accordingly. In Fig. 2, we showed simulation results of the signal to noise ratio(Eb/N0) versus bit error rate (BER) using modified PEXIT algorithm. The red cross in the Fig. 2 indicate BER of the sixteen receiver antennas at SNR value. This curve follows symmetric complex Gaussian distribution. We should generate a sufficiently large number of blocks of channel factors, i.e., a large value for $Q$.

## 4. CONCLUSION

In this paper, we have first studied the assumption of PEXIT algorithm . Then, we briefly illustrated that this assumption cannot be maintained in the case of a SIMO Rayleigh fading channel and then we elaborate how to apply the PEXIT algorithm in such an environment. To do so we propose a modified

## ACKNOWLEDGMENTS

I am thankful to my guide Assistant Professor Suman Wadkar (Pillai Institute of Information Technology, New Panvel, India) for the helpful discussions and valuable guidance.

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